

7. If the given matrix is singular, find a value for a .

$$\begin{bmatrix} 2 & a \\ a & 8 \end{bmatrix}$$

$$2k = a$$

$$ak = 8$$

$$k = \frac{a}{2} = \frac{8}{a}$$

$$a^2 = 16$$

$$a = \pm 4$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

1. Find the inverse of the given matrix, A .
2. Factor A into LU .
3. Find the transpose of A .

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & -2 \\ 0 & -3 & -5 & -2 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & -2 \\ 0 & -3 & -5 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 & 3 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & -3 & 0 & -12 & 6 & 15 \\ 0 & 0 & 1 & -2 & 1 & 3 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 4 & -2 & -5 \\ 0 & 0 & 1 & -2 & 1 & 3 \end{array} \right]$$

$$\begin{bmatrix} -1 & 1 & 1 \\ 4 & -2 & -5 \\ -2 & 1 & 3 \end{bmatrix}$$

$$2) \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$E \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -5 \\ 0 & 1 & 2 \end{bmatrix}$$

$$F \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{3} & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -5 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -5 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} U$$

$$E^T (F^T (E A)) = \underbrace{E^T F^T}_L U$$

$$E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad F^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{3} & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{3} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -\frac{1}{3} & 1 \end{bmatrix}$$

4. Factor B into LU , where $a, b, c > 0$. (5 pts)

$$B = \begin{bmatrix} a & a & a \\ a & b & b \\ a & b & c \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & a & a \\ a & b & b \\ a & b & c \end{bmatrix} = \begin{bmatrix} a & a & a \\ 0 & b-a & b-a \\ a & b & c \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & a & a \\ 0 & b-a & b-a \\ a & b & c \end{bmatrix} = \begin{bmatrix} a & a & a \\ 0 & b-a & b-a \\ 0 & b-a & c-a \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} a & a & a \\ 0 & b-a & b-a \\ 0 & b-a & c-a \end{bmatrix} = \begin{bmatrix} a & a & a \\ 0 & b-a & b-a \\ 0 & 0 & c-b \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

5. A is 2×2 matrix with $A^6 = I$. Find A , $A \neq I$.

$$\rightarrow A = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$
$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$